Diploma Programme
Programme du diplôme
Programa del Diploma

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Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 2

Tuesday 19 November 2019 (morning)

1 hour 30 minutes

Instructions to candidates

- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

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Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 15]

Casanova restaurant offers a set menu where a customer chooses **one** of the following meals: pasta, fish or shrimp.

The manager surveyed 150 customers and recorded the customer's age and chosen meal. The data is shown in the following table.

	Pasta	**Fish**	**Shrimp**	**Total**
Adults	24	25	32	81
Children	20	14	35	69
Total	44	39	67	150

A χ^2 test was performed at the 10% significance level. The critical value for this test is 4.605.

(a) State H_0, the null hypothesis for this test. [1]

(b) Write down the number of degrees of freedom. [1]

(c) Show that the expected number of children who chose shrimp is 31, correct to two significant figures. [2]

(d) Write down

(i) the χ^2 statistic;

(ii) the p-value. [3]

(e) State the conclusion for this test. Give a reason for your answer. [2]

(f) A customer is selected at random.

(i) Calculate the probability that the customer is an adult.

(ii) Calculate the probability that the customer is an adult or that the customer chose shrimp.

(iii) Given that the customer is a child, calculate the probability that they chose pasta or fish. [6]

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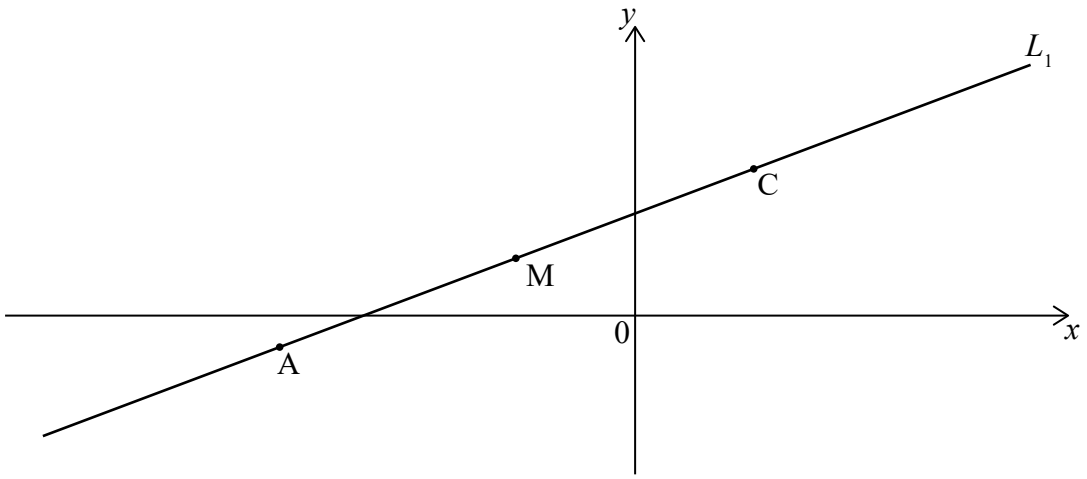
2. [Maximum mark: 13]

The diagram shows the straight line L_1. Points $\mathrm{A}(-9, -1)$, $\mathrm{M}(-3, 2)$ and C are points on L_1.



(a) Find the gradient of L_1. [2]

M is the midpoint of AC.

(b) Find the coordinates of point C. [2]

Line L_2 is perpendicular to L_1 and passes through point M.

(c) Find the equation of L_2. Give your answer in the form $ax + by + d = 0$, where a, b, $d \in \mathbb{Z}$. [3]

The point $\mathrm{N}(k, 4)$ is on L_2.

(d) Find the value of k. [2]

(e) Find the distance between points M and N. [2]

(f) Given that the length of AM is $\sqrt{45}$, find the area of triangle ANC. [2]

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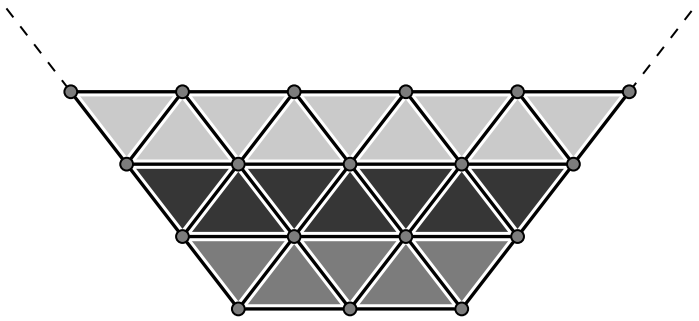
3. [Maximum mark: 15]

Maegan designs a decorative glass face for a new Fine Arts Centre. The glass face is made up of small triangular panes. The **first** three levels of the glass face are illustrated in the following diagram.

diagram not to scale



The 1st level, at the bottom of the glass face, has 5 triangular panes. The 2nd level has 7 triangular panes, and the 3rd level has 9 triangular panes. Each additional level has 2 more triangular panes than the level below it.

(a) Find the number of triangular panes in the 12th level. [3]

(b) Show that the total number of triangular panes, S_n, in the first n levels is given by:

$$S_n = n^2 + 4n\,.$$ [3]

(c) **Hence**, find the total number of panes in a glass face with 18 levels. [2]

Maegan has 1000 triangular panes to build the decorative glass face and does not want it to have any incomplete levels.

(d) Find the maximum number of **complete** levels that Maegan can build. [3]

Each triangular pane has an area of $1.84\,\text{m}^2$.

(e) Find the **total** area of the decorative glass face, if the maximum number of complete levels were built. Express your area to the nearest m^2. [4]

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4. [Maximum mark: 16]

The graph of the quadratic function $f(x) = \frac{1}{2}(x-2)(x+8)$ intersects the y-axis at $(0, c)$.

(a) Find the value of c. [2]

The vertex of the function is $(-3, -12.5)$.

(b) Write down the equation for the axis of symmetry of the graph. [2]

The equation $f(x) = 12$ has two solutions. The first solution is $x = -10$.

(c) **Use the symmetry** of the graph to show that the second solution is $x = 4$. [1]

(d) Write down the x-intercepts of the graph. [2]

(e) On graph paper, draw the graph of $y = f(x)$ for $-10 \leq x \leq 4$ and $-14 \leq y \leq 14$. Use a scale of $1\,\text{cm}$ to represent 1 unit on the x-axis and $1\,\text{cm}$ to represent 2 units on the y-axis. [4]

Let T be the tangent at $x = -3$.

(f) (i) Write down the equation of T.

(ii) Draw the tangent T on your graph. [3]

(g) Given $f(a) = 5.5$ and $f'(a) = -6$, state whether the function, f, is increasing or decreasing at $x = a$. Give a reason for your answer. [2]

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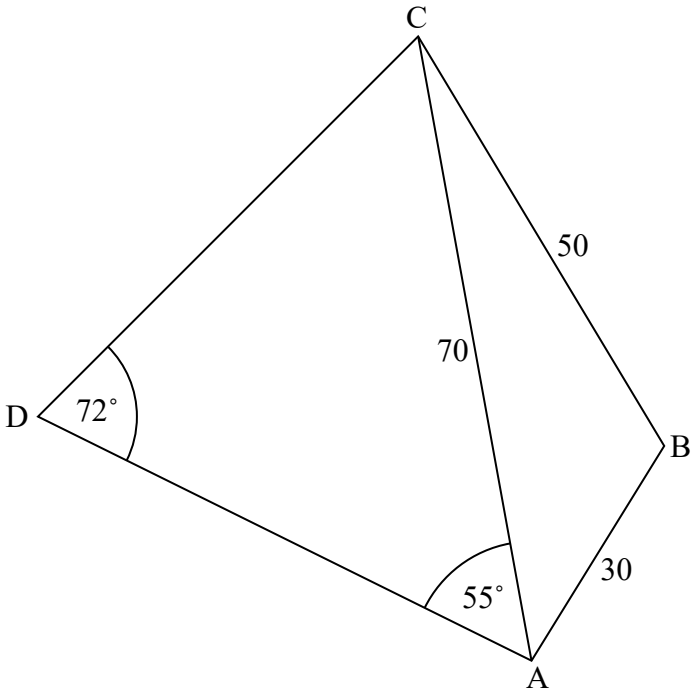
5. [Maximum mark: 15]

Haraya owns two triangular plots of land, ABC and ACD. The length of AB is 30 m, BC is 50 m and AC is 70 m. The size of $D\hat{A}C$ is 55° and $A\hat{D}C$ is 72°.

The following diagram shows this information.

diagram not to scale



(a) Find the length of AD. [4]

(b) Find the size of $A\hat{B}C$. [3]

(c) Calculate the area of the triangular plot of land ABC. [3]

(This question continues on the following page)

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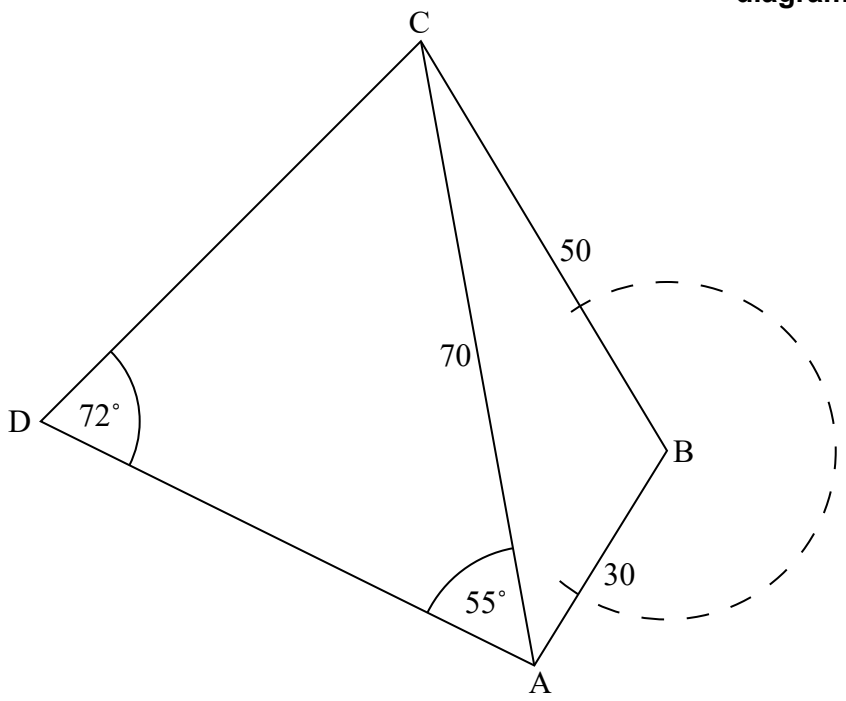
(Question 5 continued)

Haraya attaches a $20\,\text{m}$ long rope to a vertical pole at point B.

diagram not to scale



(d) Determine whether the rope can extend into the triangular plot of land, ACD. Justify your answer. [5]

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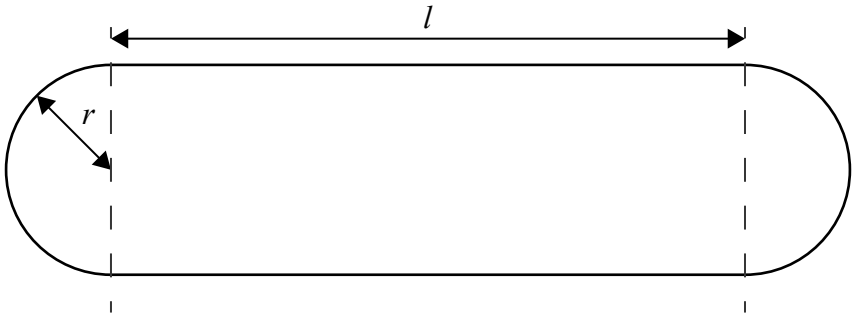
6. [Maximum mark: 16]

The Maxwell Ohm Company is designing a portable Bluetooth speaker. The speaker is in the shape of a cylinder with a hemisphere at each end of the cylinder.

The dimensions of the speaker, in centimetres, are illustrated in the following diagram where r is the radius of the hemisphere, and l is the length of the cylinder, with $r > 0$ and $l \geq 0$.



(a) Write down an expression for V, the volume (cm^3) of the speaker, in terms of r, l and π. [2]

The Maxwell Ohm Company has decided that the speaker will have a surface area of $300\,\text{cm}^2$.

(b) Write down an equation for the surface area of the speaker in terms of r, l and π. [3]

(c) Given the design constraint that $l = \dfrac{150 - 2\pi r^2}{\pi r}$, show that $V = 150r - \dfrac{2\pi r^3}{3}$. [2]

(d) Find $\dfrac{\mathrm{d}V}{\mathrm{d}r}$. [2]

The quality of sound from the speaker will improve as V increases.

(e) Using your answer to part (d), show that V is a maximum when r is equal to $\sqrt{\dfrac{75}{\pi}}\,\text{cm}$. [2]

(f) Find the length of the **cylinder** for which V is a maximum. [2]

(g) Calculate the maximum value of V. [2]

(h) Use your answer to part (f) to identify the shape of the speaker with the best quality of sound. [1]

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